Whether Insects See Anomalous Images Located Near a Lens or Not?

~ Verification of Lens Equations for Anomalous Images and Application of the Simple eye of an Insect ~

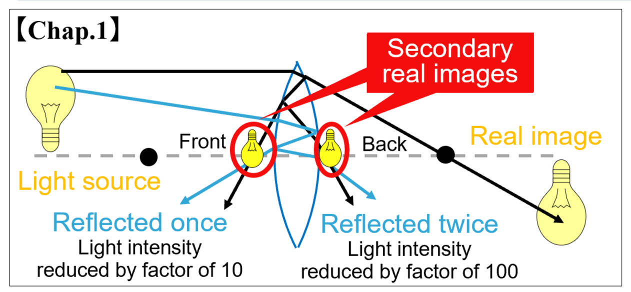
Kumamoto Prefectural Uto Junior and Senior High School

Introduction

Rainbows are caused by the reflection, refraction and dispersion of light in suspended water droplets that result in a spectrum of light that appears in the sky. We see the gathered light that has passed through those water droplets. The main rainbow is caused by light that is refracted after entering a water droplet, reflected from the back of the droplet once inside, and then refracted again when leaving it. The secondary rainbow seen outside the arc of the main rainbow results when light is reflected twice on the inside of the droplet before leaving it. This study examines "**secondary real images**" due to internal reflections within a lens, much like a rainbow is caused by reflected light in water droplets.

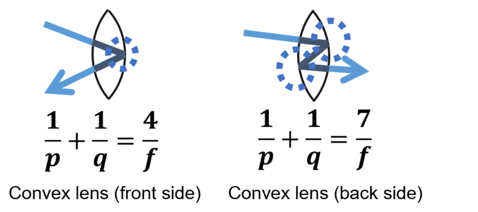
Previous Research

【Chap.1】



Previous research has shown that there are two more small images formed at the front and the back sides of a convex lens, which are known as secondary real images (donated by a SRI or SRIs). These are produced by internal reflection of light rays within a lens that is not parallel to the optical axis【Chap.1】.

【Chap.2】



SRIs have three specific features:
(1) They appear if the light source is away from the optical axis.
(2) They appear if the light source is in front of the focal point.
(3) They move in the opposite direction to the light source.

Formulae to identify the position of SRIs based on a thin convex lens model were successfully determined using a matrix approximation method【Chap.2】.

Objectives

(1) Investigate where SRIs appear for a plane-convex lens.
(2) Define the "secondary focal point" corresponding to a SRI.
(3) Investigate where SRIs appear for a convex lens whose sides have different curvature radii.
(4) Determine the location of SRIs for simple-eyed insects.

Experiments

Ⅰ. Plane-convex lens

In recent studies, the method of matrix approximation was used to determine formulae to calculate the positions where SRIs appear in a thin-model convex lens. However, **it was not clear from the calculations where secondary real images produced with a plane-convex lens would appear.** Thus, the first goal of this research is to create formulae to determine the positions of SRIs for a plane-convex lens.

Methods

The matrix approximation method is used to prove the thin lens equation reported in the textbook. In this method, we matched matrix terms. The formulae express ray penetration of a convex lens with the refraction and transmission terms used for real images【Chap.3】, and then added a reflection term for SRIs【Chap.4】. A system matrix approach was applied to SRIs for a plane-convex lens. In this method, the calculations were performed manually because it was too complicated to use Excel. We compared the experimental values and calculation results.

【Chap.3】 **Proof of the thin lens equation given in the textbook**

The process used to formulate the thin lens equation is given below using the matrix approximation.

$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix} \quad \text{-①}$$

Refraction term, Transmission term, Refraction term

$$= \begin{pmatrix} -\frac{d}{R}+\frac{d}{nR}+1 & \frac{d}{n} \\ -\frac{2n-2}{R}+\frac{d(n-2)}{R^2}+\frac{d}{nR^2} & -\frac{d}{R}+\frac{d}{nR}+1 \end{pmatrix} \quad \text{-②}$$

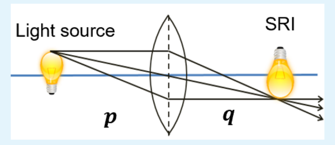
$$\fallingdotseq \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \quad \text{-③}$$

$$\begin{pmatrix} C' & D' \\ A' & B' \end{pmatrix} = \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix}\begin{pmatrix} C & D \\ A & B \end{pmatrix}\begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{q}{f} & p+q-\frac{pq}{f} \\ -\frac{1}{f} & 1-\frac{p}{f} \end{pmatrix} \quad \text{-④}$$

$$= \begin{pmatrix} 1-\frac{q}{f} & p+q-\frac{pq}{f} \\ -\frac{1}{f} & 1-\frac{p}{f} \end{pmatrix} \quad \text{-⑤}$$

$$D' = p+q-\frac{pq}{f} = 0 \quad \text{-⑥}$$

$$\frac{1}{p}+\frac{1}{q} \fallingdotseq \frac{1}{f} \quad \text{-⑦}$$



p: Distance between lens and light source
q: Distance between lens and SRI

n: Refractive index
R: Curvature radius
d: Lens thickness
f: Focal length

① Combine the terms
② Calculate ①
③ Substitute "1.5" for "n", "f" for "R", and "0" for "d" in ② (We assume the lens has no thickness)
④ Transcribe the light source which is p away from a lens to the place where is q away from a lens.
⑤ Calculate ④
⑥ Substitute 0 for D' in a point-like subject. The aberration is 0 (Image formation condition is D'= 0).
⑦ Calculate ⑥

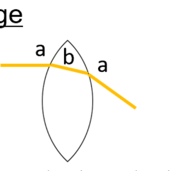
【Chap.4】 **Terms used in the formulation and the matrix terms used to describe the difference between a real image and a SRI**

$$\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ -\frac{2n}{R} & 1 \end{pmatrix}$$

a: Refraction term b: Transmission term c: Reflection term
(R: Curvature radius, n: Refractive index, d: Lens thickness)

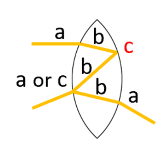
The above calculation shows the matrix terms, while the calculation below shows the differences in the way they are used. For a real image, refraction and transmission terms are used. However, for a SRI, the two terms and an additional reflection term are used in order to take into account internal reflection.

For a real image



$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} = \underset{a}{\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}}\underset{b}{\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}}\underset{a}{\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}}$$

For a SRI



$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} = \underset{a}{\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}}\underset{b}{\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}}\underset{c}{\begin{pmatrix} 1 & 0 \\ -\frac{2n}{R} & 1 \end{pmatrix}}\underset{b}{\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}}\underset{a}{\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}}$$

Four formulae for a plane-convex lens were used to locate the positions of SRIs.
Two patterns (labeled α and β) were investigated in order to check whether the calculation results were same or not. Each results were same, so we thought that we could minimize the calculation mistakes, and improve the accuracy【Chap.5】. The fact that the same results were obtained for both the α- and β-patterns confirmed the robustness of the formulae.

【Chap.5】 **Difference between α- and β-patterns**

The α- and β-patterns differ in the use of the refractive index n. In the α-pattern, n is used from the beginning of the calculation. In the β-pattern, it is not considered in the transmission or reflection term. Since the calculation results agree with each other, this acts as a double-check on the calculation.

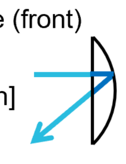
Ex.) A plane-convex lens flat side (front)

$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} = \begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{2n}{R} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix} \quad [\alpha\text{-pattern}]$$

Transmission term, Reflection term, Transmission term

$$= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{pmatrix} \quad [\beta\text{-pattern}]$$

Refraction term, Transmission term, Reflection term, Transmission term, Refraction term



Results

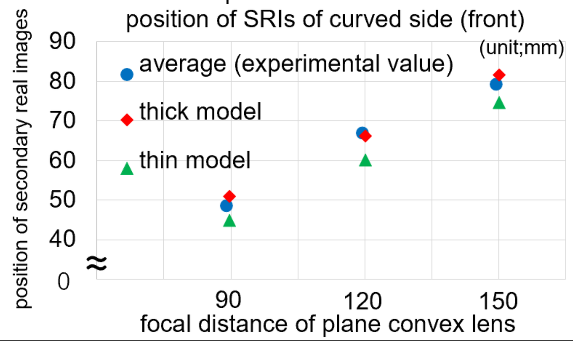
By using two matrix approximation methods, we were able to reduce calculation errors and improve the calculation accuracy. To determine whether the calculated results matched those obtained by other people, we calculated manually more than 500 times. As a result, we succeeded in creating four formulae for analyzing SRIs of a plane-convex lens【Chap.6】. The formulae were similar to the thin lens equation and the formulae for a convex lens. We compare the calculation and experimental values in【Table.1】. The calculated and experimental values were in agreement to within the experimental error.

【Table.1】 **Position of SRIs for a plane-convex lens (calculation value and experimental value*)**

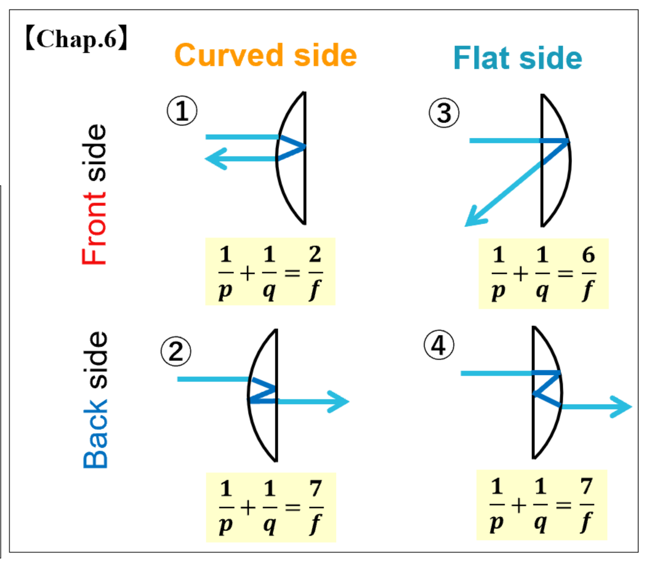
Plane-convex lens (p=10f)

		Focal length f(mm) 90	120	150
Curved side (front)	Average (experimental value)	48.4	66.6	78.6
	Thick model	51.4	66.3	81.6
	Thin model (f/2)	45.0	60.0	75.0
	Maximum error in experimental value	1.65	1.46	2.96

*We used experimental values from previous research.



【Chap.6】



Consideration

We completed four formulae for SRIs. The formulae for a plane-convex lens were similar to the thin lens equation and the formulae for a convex lens. Additionally, the formulae for the back side were all the same. From these results, we concluded that the positions of SRIs vary little with the position of the light source. In addition, the formulation proved that SRIs do not appear in the reflections of concave mirrors.

Ⅱ. Secondary focal points

According to Experiment Ⅰ, four formulae for SRIs for a plane-convex lens were completed. Next we tried to define the "secondary focal point" corresponding to SRIs. Also, we examined SRIs appear close to our life.

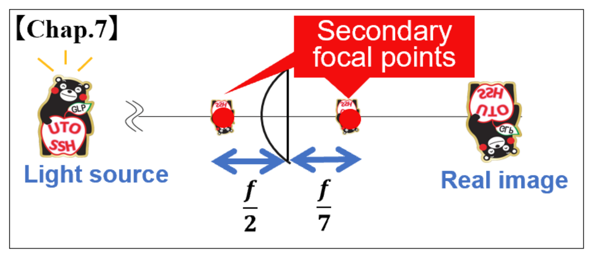
Method

SRIs are produced by internal reflections of light rays within a lens that is not parallel to the optical axis. Definition of the focal point by parallel light rays is difficult. We defined the secondary focal points as the positions of the SRIs when the light source is at an infinite distance from the lens【Chap.7】. We substituted infinity for the distance between the lens and the light source (p) in the equation we derived.

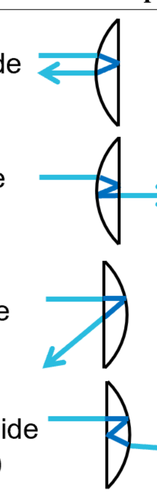
Results

We were able to define four secondary focal points of plane-convex lenses【Chap.8】.

【Chap.7】



【Chap.8】

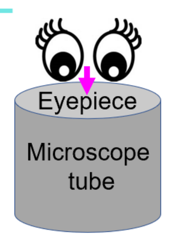


Curved side (front)	$\frac{1}{p}+\frac{1}{q}=\frac{2}{f}$	$p=\infty \Leftrightarrow q=\frac{f}{2}$
Flat side (back)	$\frac{1}{p}+\frac{1}{q}=\frac{7}{f}$	$p=\infty \Leftrightarrow q=\frac{f}{7}$
Flat side (front)	$\frac{1}{p}+\frac{1}{q}=\frac{6}{f}$	$p=\infty \Leftrightarrow q=\frac{f}{6}$
Curved side (back)	$\frac{1}{p}+\frac{1}{q}=\frac{7}{f}$	$p=\infty \Leftrightarrow q=\frac{f}{7}$

p: Distance between a lens and a light source
q: Distance between a lens and SRIs
f: Focal length

Consideration

We could determine the secondary focal point for a plane-convex lens. Also we identified that eyelashes can be seen through our eye when you look into a microscope are virtual image of SRIs. When you look into a microscope, eyelashes are illuminated by the light in the room. Then eyelashes become another light source (the position of the eyelashes are closer to the lens than the secondary focal point). Enlarged erected virtual images appear near the eyepiece and enter our field of vision.

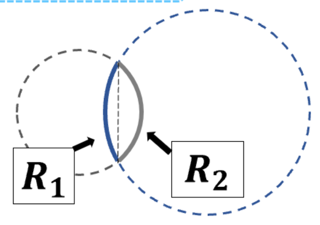


Ⅲ. Convex lens with different curvatures

In experiment Ⅰ, we succeeded in making four formulas, which were the same if applied to plane-convex lens or convex lens. Since these formulae did not cover all lenses such as a convex lens whose curvature radii are different each other, we wanted to create formulae that could be applied to all lenses.

Method

We used the matrix approximation manually and double-checked by using two methods to reduce the calculation mistakes, which were the same as before. We set the curvature radii in the front side as R_1 and in the back side as R_2 that were different each other.



Result

We calculated with checking whether the calculation results matched others. As a result, we found that a formula for the SRIs are expressed as follows.

【Chap.9】 **The formula of SRI for a convex lens which has different curvature radii (back)**

$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1-n}{R_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{2n}{R_1} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{2n}{R_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1-n}{R_1} & 1 \end{pmatrix}$$

Refraction term, Transmission term, Reflection term, Transmission term, Reflection term, Transmission term, Refraction term

$$= \begin{pmatrix} -\frac{5d}{R_1}-\frac{4d}{R_2}+\frac{4d^2}{{R_1}^2}+\frac{8d^2}{R_1R_2}-\frac{4d^3}{{R_1}^2R_2}+\frac{3d}{nR_1}-\frac{4d^2}{n{R_1}^2}-\frac{4d^2}{nR_1R_2}+\frac{4d^3}{n{R_1}^2}+1 & \frac{3d}{n}-\frac{4d^2}{nR_1}-\frac{4d^2}{nR_2}+\frac{4d^3}{nR_1R_2} \\ -\frac{3n-1}{R_1}-\frac{3n-1}{R_2}+\frac{4d(n-1)}{{R_1}^2}+\frac{4d(n-1)}{{R_2}^2}+\frac{d(11n-10)}{R_1R_2}-\frac{4d^2(2n-3)}{{R_1}^2R_2}-\frac{4d^2(2n-3)}{R_1{R_2}^2}+\frac{4d^3(n-2)}{{R_1}^2{R_2}^2}+\frac{3d}{nR_1R_2}-\frac{4d^2}{n{R_1}^2R_2}-\frac{4d^2}{nR_1{R_2}^2}+\frac{4d^3}{n{R_1}^2{R_1}^2} & -\frac{4d}{R_1}-\frac{5d}{R_2}+\frac{4d^2}{{R_2}^2}+\frac{8d^2}{R_1R_2}-\frac{4d^3}{R_1{R_2}^2}+\frac{3d}{nR_2}-\frac{4d^2}{n{R_2}^2}-\frac{4d^2}{nR_1R_2}+\frac{4d^3}{nR_1{R_2}^2}+1 \end{pmatrix}$$

Consideration

When we substituted "R" for "R_1" and "R_2", the formula matched that for a convex lens. In addition, when we substituted "R" for "R_1" and "∞" for "R_2", it matched that for a plane-convex lens curved side. When we substituted "∞" for "R_1" and "R" for "R_2", it matched that for a plane-convex lens plane side. Therefore, we were able to say that this formula applied to all lens which curvature radii were different each other.

Ⅳ. Simple eye of insects

Last year, the question of whether or not humans see SRIs was considered by analyzing the crystalline lens of a human. A human crystalline lens has an ellipsoid shape that does not cause the emergence of SRIs. Furthermore, even if they appear in a human eye, they do not fall on the retina. However, we have wondered if the eye of simple-eyed insect, which has a structure similar to that of the human eye, could possibly see SRIs. We were interested in such SRIs in the microscopic world and researched the simple eye of insects because we wondered if insects have a lens which captures the SRIs. We were interested in such SRIs in the microscopic world. In recent studies, it was determined that front side SRIs appear when light is incident on simple-eyed insects. However, the details and appearances of back-side SRIs were not determined. We experimented with Asian giant hornets, migratory locusts, and large brown cicadas【Fig.1】.

【Fig.1】 Asian giant hornet (*Vespa mandarinia japonica*), the migratory locust (*Locusta migratoria*) and the large brown cicada (*Graptopsaltria nigrofuscata*). Red circles indicate the simple eyes.

Methods

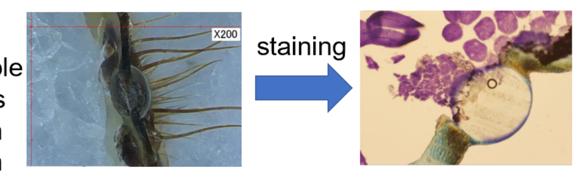
About monocular processing and finding the appearance position of SRIs.
(1) Removal of simple eyes with tweezers and observation under light microscope (Keyence)【Fig.2】.
(2) Slicing simple eyes into 10 μm thick slices with a freezing microtome (Nippon Optical Works)【Fig.3】followed by microscope observations.
(3) Staining of nucleus with Mayer's hematoxylin solution.
(4) Substituting the curvature radii, the refractive index for the formula we created.
<u>We substituted "1.5" for "n" because we supposed liquid filled the space between the lens and retina is near the physiological saline solution. We experimented on 15 Asian giant hornets and migratory locusts, as well as 30 large brown cicadas.</u>

Results

We examined lens shapes, both curvature radii (R_1,R_2), position of real images, SRIs and retinas from the lens of back side. We showed measurement and calculated values【Table.2】. We took the Asian giant hornet for the example in【Chap.10】. The SRI appears more closer to the retina than the real image.

【Chap.10】

The convex lens of simple eye which is protruded in exoskeleton



Retina cells of the back side of lens stained with purple stain

Curvature radius of front side R_1:246㎛
Curvature radius of back side R_2:132㎛

$$\begin{pmatrix} C & D \\ A & B \end{pmatrix} \quad A = -\frac{3n-1}{R_1}-\frac{3n-1}{R_2}+\frac{4d(n-1)}{{R_1}^2}+\frac{4d(n-1)}{{R_2}^2}+\frac{d(11n-10)}{R_1R_2}-\frac{4d^2(2n-3)}{{R_1}^2R_2} -\frac{4d^2(2n-3)}{R_1{R_2}^2}+\frac{4d^3(2n-3)}{{R_1}^2{R_2}^2}+\frac{3d}{nR_1R_2}-\frac{4d^2}{n{R_1}^2R_2}-\frac{4d^2}{nR_1{R_2}^2}+\frac{4d^3}{n{R_1}^2{R_2}^2}$$

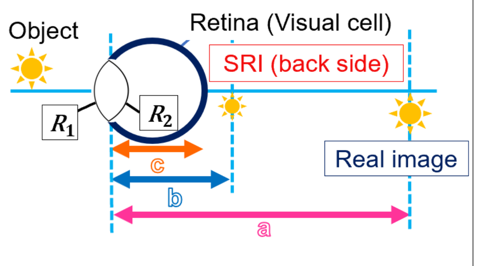
Substituting R_1=246㎛, R_2=132㎛, d=372㎛ into the formula above

The appearance position of SRI of back side
n=1.5 ⇔ 138.9(㎛)
n=1.4 ⇔ 145.9(㎛)

【Table.2】

	Lens shape	R_1	R_2	a	b	c
Asian giant hornet	Convex lens	263	192	304	138.9	100
Migratory locust	Plane-convex lens	190	∞	405	145.1	120
Large brown cicada	Convex lens	246	132	286	170.2	200

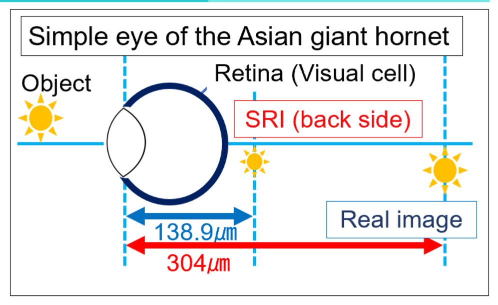
a: Appearance position of real image
b: Appearance position of SRI
c: Position of retina



<u>3 arguments to prove that insects see the SRIs</u>
① SRIs appear if the light source is away from the optical axis.
→ It is said that insects use their simple eyes to keep their balance when they are flying. So they need to see as widely as possible. Appearance angle of SRI (back side) for a convex lens is about 83.5 degrees and that for a plane-convex lens is about 180 degrees.
② Light intensity of the SRI (back side) reduced by factor of 100.
→ In order to keep their balance, insects see contrast between the sun and the ground. At that time, the sun plays a role as a light source however it is too strong. Therefore it's contemplated that the retina will burn by the strong sun light. But the light intensity of a SRI is less than original sunlight.
③ The positions of SRIs vary little with the position of the light source.
→ From previous and our researches, all formulae for SRIs were completed. According to them, it was found that the 3 formulae for SRIs (back side) were same. So appearance positions of SRIs (back side) are almost same regardless the lens shape. It means it is easy to focus SRIs on the retina.

Discussions

For the three insects examined, the SRIs appeared closer to the retina than the real image. The chapter to the right shows the position of SRIs on the back side of a simple eye of an Asian giant hornet. **<u>This suggests the possibility that insects can see SRIs, which may improve our understanding of insect physiology.</u>**



Based on this, we faced the following question.
<u>Since most insects have compound eyes, why don't the simple eyes of these insects degenerate?</u>
Multiple studies have examined compound insect eyes, but since many unknown points regarding simple-eyed insects remain, we constructed a hypothesis.

≪Hypothesis≫
1. Formation of a simple membrane helps protect the optic nerve cells.
2. The simple membrane bulges slightly to form the eye lens. (It is easy to form a spherical surface by keeping the internal pressure constant.)
3. The lens shape makes it easier to collect light and allows the thickness of the membrane to increase.
4. The thickness of the membrane increases to the point where the SRIs of the sun are caught. In this way, even if the light source is away from the optical axis, the SRIs are captured in a wide range. Since the position where the SRIs appear do not change, insects can capture these images without adjusting the membrane thickness.
5. Completion of a simple eye!

Conclusions

(1) We used the matrix approximation to created four formulae for SRIs for a plane-convex lens considered the direction, which identified the appearance positions of them. We used two different calculation methods, and the results were in good agreement.
(2) We defined the secondary focal point for the SRI, and experimentally confirmed its existence using the abovementioned formulae and tools such as microscopes.
(3) We created formulae that can be applied to lenses with different curvatures.
(4) We targeted Asian giant hornets, migratory locusts, and large brown cicadas because they have simple eyes. We then checked their lens shapes and located the SRIs using our formulae. We also identified the retina positions by staining them and found that the SRIs were closer to the retina than the real images. This suggests that insects can see SRIs.

Applications

A. Accuracy improvements for micro lenses

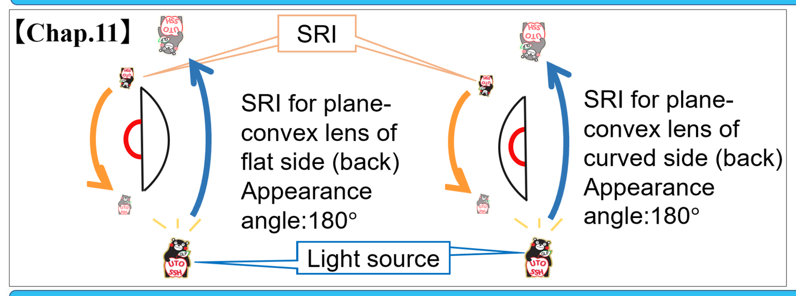
【Fig.2】 SRIs appeared on a simple eye of the Asian giant hornet (front side)

Currently, the Newton-ring method is used to check lens surfaces. However, using SRIs will allow us to easily determine whether a lens has a spherical surface because these images only appear on convex and plane-convex lenses. (Not on aspheric or concave lenses.) Thus, using SRIs will make it unnecessary to process a lens and it can be observed without touching it physically. It is like remote sensing* in microscopic world!! SRIs will be especially useful for evaluating micro lenses as small as those of simple-eyed insects【Fig.2】.

*Remote sensing : It is technology that to check a thing without touching it. It is represented by artificial satellite and aircraft.

B. Wide-angle lens development

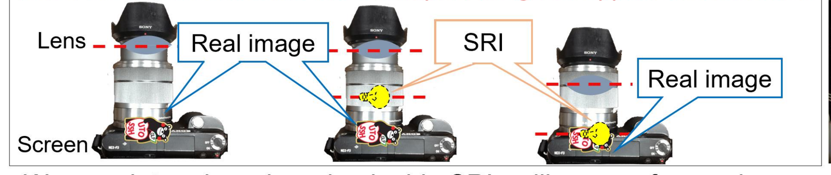
【Chap.11】

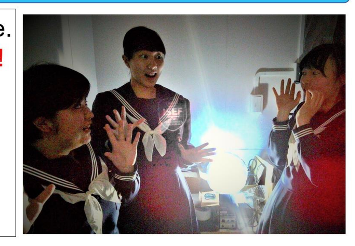


SRIs can be used to capture a wide angular range because their appearance angle for a plane-convex lens is 180°. This means, for example, that we can easily capture the movement of meteorites or meteor showers.

C. Preventing or making ghost photographs

【Chap.12】 This is a mechanism how such ghost photograph are made. When the screen and SRIs are overlap…, the ghost appear on a screen!!





We can determine where back-side SRIs will appear for any lens by using the formulae we created. This means we can take ghost photographs by adjusting the distance between the lens and the screen. We can also prevent ghost photographs by using anti-coating lenses because it was found that the SRIs are formed by internal reflections within a lens【Chap.12】【Fig.3】.

【Fig.3】 This is a ghost photograph created by authors!! You can see the logo of the Intel ISEF!

References

[1] Publication Center for Pure and Applied Physics, Journal of the Physical Society of Japan, 68_2013.3,（27J-8）
[2] Eugene Hecht, "Optics: International Edition"
[3] K.Kuroda,"Optics Chapter 1 Geometric Optics" http://qopt.iis.u-Tokyo.ac.jp/optics/3lensU_A4.pdf
[4] T.Yagi,"Geometric Optics Chapter 1" http://www.sp.u-tokai.ac.jp/~yagi/OpricsandLaser
[5] S.Kano,"Ray tracing by Geometric Optics" http://cis.khosei.ac.jp/~kanoopt/opt_01.pdf
[6] Raymond A.Serway, Jerry S. Faughn, "Physics -Amazing small brain-"
[7] M.Mizunami, "Insects"
☆All Figures were made by us.